

## 10.4 Videos Guide

### 10.4a

- Area in polar coordinates
  - $A = \int_a^b \frac{1}{2} [f(\theta)]^2 d\theta$
  - Between two curves  $f$  and  $g$  ( $f \geq g$ ):  $A = \frac{1}{2} \int_a^b \{[f(\theta)]^2 - [g(\theta)]^2\} d\theta$
- Arc length
  - $L = \int_a^b \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} d\theta = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$

Exercises:

### 10.4b

- Graph the curve and find the area that it encloses.  
 $r = 3 - 2 \cos 4\theta$

### 10.4c

- Find the area of the region that lies inside the first curve and outside the second curve.  
 $r = 3 \sin \theta, \quad r = 2 - \sin \theta$

### 10.4d

- Find the area of the region that lies inside both curves.  
 $r = \sin 2\theta, \quad r = \cos 2\theta$

### 10.4e

- Find the length of the polar curve.  
 $r = 5^\theta, \quad 0 \leq \theta \leq 2\pi$

### 10.4f

- Tangents to a polar curve: if  $r = f(\theta)$  and  $x = r \cos \theta, \quad y = r \sin \theta$ , then
  - $\frac{dy}{d\theta} = \frac{dr}{d\theta} \sin \theta + r \cos \theta$  (by the product rule)
  - $\frac{dx}{d\theta} = \frac{dr}{d\theta} \cos \theta - r \sin \theta$  (product rule again)
  - $\frac{dy}{dx} = \frac{\frac{dr}{d\theta} \sin \theta + r \cos \theta}{\frac{dr}{d\theta} \cos \theta - r \sin \theta}$
  - Horizontal tangents occur when  $\frac{dy}{d\theta} = 0$  (if  $\frac{dx}{d\theta} \neq 0$ )
  - Vertical tangents occur when  $\frac{dx}{d\theta} = 0$  (if  $\frac{dy}{d\theta} \neq 0$ )

Note: if both  $\frac{dy}{d\theta} = 0$  and  $\frac{dx}{d\theta} = 0$ , we use l'Hospital's Rule

Exercises:

- Find the slope of the tangent line to the given polar curve at the point specified by the value of  $\theta$ .

$$r = 2 + \sin 3\theta, \quad \theta = \pi/4$$

10.4g

- Find the points on the given curve where the tangent line is horizontal or vertical.

$$r = 1 - \sin \theta$$